Chapter 18

Mehar method for solving unbalanced generalized interval-valued trapezoidal fuzzy number transportation problems

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1 Introduction

Ebrahimnejad (2016) pointed out that several methods have been proposed for solving fuzzy/fully fuzzy transportation problems (transportation problems in which each parameter is represented by a triangular/trapezoidal fuzzy number) (Ebrahimnejad, 2014, 2015a,b,c, 2016; Oheigeartaigh, 1982; Chanas et al., 1984, 1993; Chanas and Kuchta, 1996; Jimenez and Verdegay, 1998, 1999; Liu and Kao, 2004; Dinagar and Palanivel, 2009; Pandian and Natarajan, 2010; Kumar and Kaur, 2010, 2011a,b, 2014; Gupta et al., 2012; Shanmugasundari and Ganesan, 2013; Sudhagar and Ganesan, 2012; Kaur and Kumar, 2012; Chiang, 2005; Gupta and Kumar, 2012). However, no method has been proposed to find the solution of generalized interval-valued fuzzy transportation problems (transportation problems in which each parameter is represented by a triangular/trapezoidal fuzzy number). To fill this gap, Ebrahimnejad (2016) proposed the following methods:

1. A method to transform an unbalanced generalized IVTrFTP into a balanced generalized IVTrFTP.
2. A linear programming method to find the solution of a balanced generalized IVTrFTP.

Ebrahimnejad (2016) considered an unbalanced generalized IVTrFNTP and applied the methods, proposed by him, to transform it into a balanced generalized IVTrFNTP as well as to find its solution.

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In this paper, it is shown that on applying Ebrahimnejad's method for transforming an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP (Ebrahimnejad, 2016), the obtained dummy supply and/or dummy demand is not a generalized interval-valued trapezoidal fuzzy number (IVTrFN), and therefore this method is not valid. In addition, a new method (known as the Mehar method) is proposed to transform an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP. Furthermore, the validity of the proposed Mehar method is discussed.

This paper is organized as follows:

2 Ebrahimnejad's method for transforming an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP

Ebrahimnejad proposed the following method to transform an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP, i.e., to transform $\sum_{i=1}^{m}\tilde{\tilde{a}}_i \neq \sum_{j=1}^{n}\tilde{\tilde{b}}_j$ into $\sum_{i=1}^{m}\tilde{\tilde{a}}_i = \sum_{j=1}^{n}\tilde{\tilde{b}}_j$, where, m represents number of sources, n represents number of destinations,

$$\sum_{i=1}^{m}\tilde{\tilde{a}}_i = \left(\left(\sum_{i=1}^{m}a_{i1}^L, \sum_{i=1}^{m}a_{i2}^L, \sum_{i=1}^{m}a_{i3}^L, \sum_{i=1}^{m}a_{i4}^L;\ \omega^L\right), \left(\sum_{i=1}^{m}a_{i1}^U, \sum_{i=1}^{m}a_{i2}^U, \sum_{i=1}^{m}a_{i3}^U, \sum_{i=1}^{m}a_{i4}^U;\ \omega^U\right)\right)$$

represents total interval-valued fuzzy supply, and

$$\sum_{j=1}^{n}\tilde{\tilde{b}}_j = \left(\left(\sum_{j=1}^{n}b_{j1}^L, \sum_{j=1}^{n}b_{j2}^L, \sum_{j=1}^{n}b_{j3}^L, \sum_{j=1}^{n}b_{j4}^L;\ \omega^L\right), \left(\sum_{j=1}^{n}b_{j1}^U, \sum_{j=1}^{n}b_{j2}^U, \sum_{j=1}^{n}b_{j3}^U, \sum_{j=1}^{n}b_{j4}^U;\ \omega^U\right)\right)$$

represents total interval-valued fuzzy demand. The generalized IVTrFN $\langle(a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; \omega^L), (a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; \omega^U)\rangle$ represents the supply of the product at ith source S_i. The generalized IVTrFN $\langle(b_{j1}^L, b_{j2}^L, b_{j3}^L, b_{j4}^L; \omega^L), (b_{j1}^U, b_{j2}^U, b_{j3}^U, b_{j4}^U; \omega^U)\rangle$ represents the demand of the product at jth destination D_j.

Case 1: If $\sum_{i=1}^{m}a_{i1}^L \le \sum_{j=1}^{n}b_{j1}^L$, $\sum_{i=1}^{m}a_{i2}^L \le \sum_{j=1}^{n}b_{j2}^L$, $\sum_{i=1}^{m}a_{i3}^L \le \sum_{j=1}^{n}b_{3}^L$, $\sum_{i=1}^{m}a_{i4}^L \le \sum_{j=1}^{n}b_{j4}^L$, $\sum_{i=1}^{m}a_{i1}^U \le \sum_{j=1}^{n}b_{j1}^U$, $\sum_{i=1}^{m}a_{i2}^U \le \sum_{j=1}^{n}b_{j2}^U$, $\sum_{i=1}^{m}a_{i3}^U \le \sum_{j=1}^{n}b_{j3}^U$, $\sum_{i=1}^{m}a_{i4}^U \le \sum_{j=1}^{n}b_{j4}^U$, then add a dummy source S_{m+1} having dummy supply

$$\left\langle \begin{array}{l}\left(\sum_{j=1}^{n}b_{j1}^L - \sum_{i=1}^{m}a_{i1}^L, \sum_{j=1}^{n}b_{j2}^L - \sum_{i=1}^{m}a_{i2}^L, \sum_{j=1}^{n}b_{j3}^L - \sum_{i=1}^{m}a_{i3}^L, \sum_{j=1}^{n}b_{j4}^L - \sum_{i=1}^{m}a_{i4}^L;\ \omega^L\right), \\ \left(\sum_{j=1}^{n}b_{j1}^U - \sum_{i=1}^{m}a_{i1}^U, \sum_{j=1}^{n}b_{j2}^U - \sum_{i=1}^{m}a_{i2}^U, \sum_{j=1}^{n}b_{j3}^U - \sum_{i=1}^{m}a_{i3}^U, \sum_{j=1}^{n}b_{j4}^U - \sum_{i=1}^{m}a_{i4}^U;\ \omega^U\right)\end{array}\right\rangle$$

by considering the cost for supplying the unit quantity of the product from the dummy source S_{m+1} to all the destinations as a generalized IVTrFN $\tilde{\tilde{0}} = \left(\left(0,0,0,0;\,1\right),\left(0,0,0,0;\,1\right)\right)$.

Case 2: If $\sum_{j=1}^{n} b_{j1}^{L} \le \sum_{i=1}^{m} a_{i1}^{L}$, $\sum_{j=1}^{n} b_{j2}^{L} \le \sum_{i=1}^{m} a_{i2}^{L}$, $\sum_{j=1}^{n} b_{j3}^{L} \le \sum_{i=1}^{m} a_{i3}^{L}$, $\sum_{j=1}^{n} b_{j4}^{L} \le \sum_{i=1}^{m} a_{i4}^{L}$, $\sum_{j=1}^{n} b_{j1}^{U} \le \sum_{i=1}^{m} a_{i1}^{U}$, $\sum_{j=1}^{n} b_{j2}^{U} \le \sum_{i=1}^{m} a_{i2}^{U}$, $\sum_{j=1}^{n} b_{j3}^{U} \le \sum_{i=1}^{m} a_{i3}^{U}$, $\sum_{j=1}^{n} b_{j4}^{U} \le \sum_{i=1}^{m} a_{i4}^{U}$, then add a dummy destination D_{n+1} having dummy demand

$$\left\langle \begin{array}{l} \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{j=1}^{n} b_{j1}^{L}, \sum_{i=1}^{m} a_{i2}^{L} - \sum_{j=1}^{n} b_{j2}^{L}, \sum_{i=1}^{m} a_{i3}^{L} - \sum_{j=1}^{n} b_{j3}^{L}, \sum_{i=1}^{m} a_{i4}^{L} - \sum_{j=1}^{n} b_{j4}^{L};\, \omega^{L}\right), \\ \left(\sum_{i=1}^{m} a_{i1}^{U} - \sum_{j=1}^{n} b_{j1}^{U}, \sum_{i=1}^{m} a_{i2}^{U} - \sum_{j=1}^{n} b_{j2}^{U}, \sum_{i=1}^{m} a_{i3}^{U} - \sum_{j=1}^{n} b_{j3}^{U}, \sum_{i=1}^{m} a_{i4}^{U} - \sum_{j=1}^{n} b_{j4}^{U};\, \omega^{U}\right) \end{array} \right\rangle$$

by considering the cost for supplying the unit quantity of the product from the dummy destination D_{n+1} to all the sources as a generalized IVTrFN, $\tilde{\tilde{0}} = \left(\left(0,0,0,0;1\right),\left(0,0,0,0;1\right)\right)\cdot$

Case 3: If neither Case 1 nor Case 2 is satisfied, then carry out the following:

(i) Add a dummy source S_{m+1} having the dummy supply $\left\langle \left(A_{(m+1)1}^{L}, A_{(m+1)2}^{L}, A_{(m+1)3}^{L}, A_{(m+1)4}^{L}\right), \left(A_{(m+1)1}^{U}, A_{(m+1)2}^{U}, A_{(m+1)3}^{U}, A_{(m+1)4}^{U}\right)\right\rangle$ by considering the cost for supplying the unit quantity of the product from all the dummy sources S_{m+1} to all the dummy destinations as generalized IVTrFN $\tilde{\tilde{0}} = \left(\left(0,0,0,0;1\right),\left(0,0,0,0;1\right)\right)$

$$A_{(m+1)1}^{L} = \left|\sum_{j=1}^{n} b_{j1}^{U} - \sum_{i=1}^{m} a_{i1}^{U}\right| + \max\left\{0, \sum_{j=1}^{n} b_{j1}^{L} - \sum_{i=1}^{m} a_{i1}^{L}\right\},$$

$$A_{(m+1)2}^{L} = A_{(m+1)1}^{L} + \max\left\{0, \left(\sum_{j=1}^{n} b_{j2}^{L} - \sum_{j=1}^{n} b_{j1}^{L}\right) - \left(\sum_{i=1}^{m} a_{i2}^{L} - \sum_{i=1}^{m} a_{i1}^{L}\right)\right\},$$

$$A_{(m+1)3}^{L} = A_{(m+1)2}^{L} + \max\left\{0, \left(\sum_{j=1}^{n} b_{j3}^{L} - \sum_{j=1}^{n} b_{j2}^{L}\right) - \left(\sum_{i=1}^{m} a_{i3}^{L} - \sum_{i=1}^{m} a_{i2}^{L}\right)\right\},$$

$$A_{(m+1)4}^{L} = A_{(m+1)3}^{L} + \max\left\{0, \left(\sum_{j=1}^{n} b_{j4}^{L} - \sum_{j=1}^{n} b_{j3}^{L}\right) - \left(\sum_{i=1}^{m} a_{i4}^{L} - \sum_{i=1}^{m} a_{i3}^{L}\right)\right\},$$

$$A_{(m+1)1}^{U} = \max\left\{0, \sum_{j=1}^{n} b_{j1}^{U} - \sum_{i=1}^{m} a_{i1}^{U}\right\},$$

$$A^U_{(m+1)2}=\sum_{j=1}^{n}b^U_{j1}-\sum_{i=1}^{m}a^U_{i1}+\max\left\{0,\sum_{j=1}^{n}b^U_{j1}-\sum_{i=1}^{m}a^U_{i1}\right\}$$
$$+\max\left\{0,\left(\sum_{j=1}^{n}b^U_{j2}-\sum_{j=1}^{n}b^U_{j1}\right)-\left(\sum_{i=1}^{m}a^U_{i2}-\sum_{i=1}^{m}a^U_{i1}\right)\right\},$$

$$A^U_{(m+1)3}=A^U_{(m+1)2}+\max\left\{0,\left(\sum_{j=1}^{n}b^U_{j3}-\sum_{j=1}^{n}b^U_{j2}\right)-\left(\sum_{i=1}^{m}a^U_{i3}-\sum_{i=1}^{m}a^U_{i2}\right)\right\},$$

$$A^U_{(m+1)4}=A^U_{(m+1)3}+\max\left\{0,\left(\sum_{j=1}^{n}b^U_{j4}-\sum_{j=1}^{n}b^U_{j3}\right)-\left(\sum_{i=1}^{m}a^U_{i4}-\sum_{i=1}^{m}a^U_{i3}\right)\right\}.$$

(ii) Add a dummy destination D_{n+1} having the dummy demand $\left\langle(B^L_{(n+1)1}, B^L_{(n+1)2}, B^L_{(n+1)3}, B^L_{(n+1)4}), (B^U_{(n+1)1}, B^U_{(n+1)2}, B^U_{(n+1)3}, B^U_{(n+1)4})\right\rangle$ by considering the cost for supplying the unit quantity of the product from all the sources to the dummy destination D_{n+1} as generalized IVTrFN $\tilde{\tilde{0}}=\left((0,0,0,0;1),(0,0,0,0;1)\right)\cdot$

$$B^L_{(n+1)1}=\left|\sum_{j=1}^{n}b^U_{j1}-\sum_{i=1}^{m}a^U_{i1}\right|+\max\left\{0,\sum_{i=1}^{m}a^L_{i1}-\sum_{j=1}^{n}b^L_{j1}\right\},$$

$$B^L_{(n+1)2}=B^L_{(n+1)1}+\max\left\{0,\left(\sum_{i=1}^{m}a^L_{i2}-\sum_{i=1}^{m}a^L_{i1}\right)-\left(\sum_{j=1}^{n}b^L_{j2}-\sum_{j=1}^{n}b^L_{j1}\right)\right\},$$

$$B^L_{(n+1)3}=B^L_{(n+1)2}+\max\left\{0,\left(\sum_{i=1}^{m}a^L_{i3}-\sum_{i=1}^{m}a^L_{i2}\right)-\left(\sum_{j=1}^{n}b^L_{j3}-\sum_{j=1}^{n}b^L_{j2}\right)\right\},$$

$$B^L_{(n+1)4}=B^L_{(n+1)3}+\max\left\{0,\left(\sum_{i=1}^{m}a^L_{i4}-\sum_{i=1}^{m}a^L_{i3}\right)-\left(\sum_{j=1}^{n}b^L_{j4}-\sum_{j=1}^{n}b^L_{j3}\right)\right\},$$

$$B^U_{(n+1)1}=\max\left\{0,\sum_{i=1}^{m}a^U_{i1}-\sum_{j=1}^{n}b^U_{j1}\right\},$$

$$B^U_{(n+1)2}=\left|\sum_{i=1}^{m}a^U_{i1}-\sum_{j=1}^{n}b^U_{j1}\right|+\max\left\{0,\sum_{i=1}^{m}a^U_{i1}-\sum_{j=1}^{n}b^U_{j1}\right\}$$
$$+\max\left\{0,\left(\sum_{i=1}^{m}a^U_{i2}-\sum_{i=1}^{m}a^U_{i1}\right)-\left(\sum_{j=1}^{n}b^U_{j2}-\sum_{j=1}^{n}b^U_{j1}\right)\right\},$$

$$B^U_{(n+1)3} = B^U_{(n+1)2} + \max\left\{0, \left(\sum_{i=1}^{m} a^U_{i3} - \sum_{i=1}^{m} a^U_{i2}\right) - \left(\sum_{j=1}^{n} b^U_{j3} - \sum_{j=1}^{n} b^U_{j2}\right)\right\},$$

$$B^U_{(n+1)4} = B^U_{(n+1)3} + \max\left\{0, \left(\sum_{i=1}^{m} a^U_{i4} - \sum_{i=1}^{m} a^U_{i3}\right) - \left(\sum_{j=1}^{n} b^U_{j4} - \sum_{j=1}^{n} b^U_{j3}\right)\right\}.$$

3 Flaws of Ebrahimnejad's method for transforming an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP

In this section, some numerical values are considered to show that Ebrahimnejad's method for transforming an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP (Ebrahimnejad, 2016) is not valid.

(1) Let $\left\langle\left(\sum_{i=1}^{m} a^L_{i1}, \sum_{i=1}^{m} a^L_{i2}, \sum_{i=1}^{m} a^L_{i3}, \sum_{i=1}^{m} a^L_{i4};\ \omega^L\right), \left(\sum_{i=1}^{m} a^U_{i1}, \sum_{i=1}^{m} a^U_{i2}, \sum_{i=1}^{m} a^U_{i3}, \sum_{i=1}^{m} a^U_{i4};\ \omega^U\right)\right\rangle = \langle(1,2,7,11;1),(0,3,8,7;1)\rangle$ and $\left\langle\left(\sum_{j=1}^{n} b^L_{j1}, \sum_{j=1}^{n} b^L_{j2}, \sum_{j=1}^{n} b^L_{j3}, \sum_{j=1}^{n} b^L_{j4};\ \omega^L\right), \left(\sum_{j=1}^{n} b^U_{j1}, \sum_{j=1}^{n} b^U_{j2}, \sum_{j=1}^{n} b^U_{j3}, \sum_{j=1}^{n} b^U_{j4};\ \omega^U\right)\right\rangle = \langle(4,8,9,13;1),(1,6,10,18;1)\rangle$.

Since $\sum_{i=1}^{m} a^L_{i1} \le \sum_{j=1}^{n} b^L_{j1}$, $\sum_{i=1}^{m} a^L_{i2} \le \sum_{j=1}^{n} b^L_{j2}$, $\sum_{i=1}^{m} a^L_{i3} \le \sum_{j=1}^{n} b^L_{j3}$, $\sum_{i=1}^{m} a^L_{i4} \le \sum_{j=1}^{n} b^L_{j4}$, $\sum_{i=1}^{m} a^U_{i1} \le \sum_{j=1}^{n} b^U_{j1}$, $\sum_{i=1}^{m} a^U_{i2} \le \sum_{j=1}^{n} b^U_{j2}$, $\sum_{i=1}^{m} a^U_{i3} \le \sum_{j=1}^{n} b^U_{j3}$, $\sum_{i=1}^{m} a^U_{i4} \le \sum_{j=1}^{n} b^U_{j4}$, i.e., Case 1 of Ebrahimnejad's method (Ebrahimnejad, 2016), discussed in Section 2, is satisfied. So, according to (Ebrahimnejad's (2016) method, the dummy supply will be $\langle(4-1, 8-2, 9-7, 13-11; 1), (1-0, 6-3, 10-8, 18-7; 1)\rangle = \langle(3,6,2,2;1),(1,3,2,11;1)\rangle$.

However, it is not a generalized IVTrFN for the following reason: It can be easily verified from the graphical representation of generalized IVTrFN (Ebrahimnejad, 2016) as well as from the existing definition of a generalized IVTrFN (Ebrahimnejad, 2016) that in a generalized IVTrFN $\langle(a_1{}^L, a_2{}^L, a_3{}^L, a_4{}^L; \omega^L), (a_1{}^U, a_2{}^U, a_3{}^U, a_4{}^U; \omega^U)\rangle$ the condition $a_1{}^U \le a_1{}^L \le a_2{}^U \le a_2{}^L \le a_3{}^L \le a_3{}^U \le a_4{}^L \le a_4{}^U$ should always be satisfied. While, for the obtained dummy supply $\langle(3,6,2,2;1),(1,3,2,11;1)\rangle$, this condition is not satisfied. Hence, Ebrahimnejad's method (Ebrahimnejad, 2016) to obtain the dummy supply is not correct.

(2) Let $\left\langle\left(\sum_{i=1}^{m}a_{i1}^{L},\sum_{i=1}^{m}a_{i2}^{L},\sum_{i=1}^{m}a_{i3}^{L},\sum_{i=1}^{m}a_{i4}^{L};\,\omega^{L}\right),\left(\sum_{i=1}^{m}a_{i1}^{U},\sum_{i=1}^{m}a_{i2}^{U},\sum_{i=1}^{m}a_{i3}^{U},\sum_{i=1}^{m}a_{i4}^{U};\,\omega^{U}\right)\right\rangle$ and $=\langle(4,8,9,13;1),(1,6,10,18;1)\rangle$

$\left\langle\left(\left(\sum_{j=1}^{n}b_{j1}^{L},\sum_{j=1}^{n}b_{j2}^{L},\sum_{j=1}^{n}b_{j3}^{L},\sum_{j=1}^{n}b_{j4}^{L};\,\omega^{L}\right),\left(\sum_{j=1}^{n}b_{j1}^{U},\sum_{j=1}^{n}b_{j2}^{U},0\sum_{j=1}^{n}b_{j3}^{U},\sum_{j=1}^{n}b_{j4}^{U};\,\omega^{U}\right)\right\rangle$ $=\langle(1,2,7,11;1),(0,3,8,7;1)\rangle$.

Since $\sum_{j=1}^{n}b_{j1}^{L}\le\sum_{i=1}^{m}a_{i1}^{L}$, $\sum_{j=1}^{n}b_{j2}^{L}\le\sum_{i=1}^{m}a_{i2}^{L}$, $\sum_{j=1}^{n}b_{j3}^{L}\le\sum_{i=1}^{m}a_{i3}^{L}$, $\sum_{j=1}^{n}b_{j4}^{L}\le\sum_{i=1}^{m}a_{i4}^{L}$, $\sum_{j=1}^{n}b_{j1}^{U}\le\sum_{i=1}^{m}a_{i1}^{U}$, $\sum_{j=1}^{n}b_{j2}^{U}\le\sum_{i=1}^{m}a_{i2}^{U}$, $\sum_{j=1}^{n}b_{j3}^{U}\le\sum_{i=1}^{m}a_{i3}^{U}$, $\sum_{j=1}^{n}b_{j4}^{U}\le\sum_{i=1}^{m}a_{i4}^{U}$, i.e., Case 2 of Ebrahimnejad's method (Ebrahimnejad, 2016), discussed in Section 2, is satisfied. So, according to Ebrahimnejad's method (Ebrahimnejad, 2016), the dummy demand will be $\langle(4-1,8-2,9-7,13-11;1),(1-0,6-3,10-8,18-7;1)\rangle=\langle(3,6,2,2;1),(1,3,2,11;1)\rangle$.

However, it is not a generalized IVTrFN for the following reason. It can be easily verified from the graphical representation of generalized IVTrFN (Ebrahimnejad, 2016) as well as from the existing definition of a generalized IVTrFN (Ebrahimnejad, 2016) that in a generalized IVTrFN $\langle({a_1}^{L},{a_2}^{L},{a_3}^{L},{a_4}^{L};\omega^{L}),({a_1}^{U},{a_2}^{U},{a_3}^{U},{a_4}^{U};\omega^{U})\rangle$ the condition ${a_1}^{U}\le{a_1}^{L}\le{a_2}^{U}\le{a_2}^{L}\le{a_3}^{L}\le{a_3}^{U}\le{a_4}^{L}\le{a_4}^{U}$ should always be satisfied. While, for the obtained dummy demand $\langle(3,6,2,2;1),(1,3,2,11;1)\rangle$, this condition is not satisfied. Hence, Ebrahimnejad's method (Ebrahimnejad, 2016) to obtain the dummy demand is not correct.

(3) Let $\left\langle\left(\sum_{i=1}^{m}a_{i1}^{L},\sum_{i=1}^{m}a_{i2}^{L},\sum_{i=1}^{m}a_{i3}^{L},\sum_{i=1}^{m}a_{i4}^{L};\,\omega^{L}\right),\left(\sum_{i=1}^{m}a_{i1}^{U},\sum_{i=1}^{m}a_{i2}^{U},\sum_{i=1}^{m}a_{i3}^{U},\sum_{i=1}^{m}a_{i4}^{U};\,\omega^{U}\right)\right\rangle$ $=\left\langle\left(110,150,160,180;\,\frac{2}{3}\right),(100,140,170,190;\,1)\right\rangle$

and $\left\langle\left(\sum_{j=1}^{n}b_{j1}^{L},\sum_{j=1}^{n}b_{j2}^{L},\sum_{j=1}^{n}b_{j3}^{L},\sum_{j=1}^{n}b_{j4}^{L};\,\omega^{L}\right),\left(\sum_{j=1}^{n}b_{j1}^{U},\sum_{j=1}^{n}b_{j2}^{U},\sum_{j=1}^{n}b_{j3}^{U},\sum_{j=1}^{n}b_{j4}^{U};\,\omega^{U}\right)\right\rangle$ $=\left\langle\left(90,120,140,200;\frac{2}{3}\right),(75,105,155,215;1)\right\rangle$.

Since $\sum_{j=1}^{n}b_{j1}^{L}\le\sum_{i=1}^{m}a_{i1}^{L}$, $\sum_{j=1}^{n}b_{j2}^{L}\le\sum_{i=1}^{m}a_{i2}^{L}$, $\sum_{j=1}^{n}b_{j3}^{L}\le\sum_{i=1}^{m}a_{i3}^{L}$, $\sum_{i=1}^{m}a_{i4}^{L}\le\sum_{j=1}^{n}b_{j4}^{L}$, $\sum_{j=1}^{n}b_{j1}^{U}\le\sum_{i=1}^{m}a_{i1}^{U}$, $\sum_{j=1}^{n}b_{j2}^{U}\le\sum_{i=1}^{m}a_{i2}^{U}$, $\sum_{j=1}^{n}b_{j3}^{U}\le\sum_{i=1}^{m}a_{i3}^{U}$, $\sum_{i=1}^{m}a_{i4}^{U}\le\sum_{j=1}^{n}b_{j4}^{U}$, i.e., Case 3 of

Ebrahimnejad's method (Ebrahimnejad, 2016), discussed in Section 2, is satisfied. So, according to Ebrahimnejad's method (Ebrahimnejad, 2016), the dummy demand will be $\left\langle\left(45,55,55,55;\,\frac{2}{3}\right),\left(25,60,60,60;\,1\right)\right\rangle$.

However, the obtained dummy demand is not a generalized IVTrFN for the following reason. It can be easily verified from the graphical representation of generalized IVTrFN (Ebrahimnejad, 2016) as well as from the existing definition of a generalized IVTrFN (Ebrahimnejad, 2016) that in a generalized IVTrFN $\langle(a_1^L,a_2^L,a_3^L,a_4^L;\omega^L),(a_1^U,a_2^U,a_3^U,a_4^U;\omega^U)\rangle$ the condition $a_1^U\le a_1^L\le a_2^U\le a_2^L\le a_3^L\le a_3^U\le a_4^L\le a_4^U$ should always be satisfied. While, for the obtained dummy demand $\left\langle\left(45,55,55,55;\frac{2}{3}\right),\left(25,60,60,60;1\right)\right\rangle$, this condition is not satisfied. Hence, Ebrahimnejad's method (Ebrahimnejad, 2016) to obtain the dummy demand is not correct.

4 Proposed Mehar method for transforming an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP

In this section, a new method (known as the Mehar method) is proposed to transform an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP.

Let us consider an unbalanced generalized IVTrFNTP having m sources and n destinations such that the availability of the product and the demand of the product at ith source (S_i) and jth destination (D_j) be represented by generalized IVTrFNTP $\tilde{\tilde{a}}_i=\left\langle\left(a_{i1}^L,a_{i2}^L,a_{i3}^L,a_{i4}^L;\,\omega^L\right),\left(a_{i1}^U,a_{i2}^U,a_{i3}^U,a_{i4}^U;\,\omega^U\right)\right\rangle$ and $\tilde{\tilde{b}}_j=\left\langle\left(b_{j1}^L,b_{j2}^L,b_{j3}^L,b_{j4}^L;\,\omega^L\right),\left(b_{j1}^U,b_{j2}^U,b_{j3}^U,b_{j4}^U;\,\omega^U\right)\right\rangle$, respectively. Then, this unbalanced generalized IVTrFNTP can be transformed into a balanced generalized IVTrFNTP as follows:

Case 1: If $\sum_{i=1}^m a_{i1}^U\le\sum_{j=1}^n b_{j1}^U$, $\sum_{i=1}^m a_{i1}^L-\sum_{i=1}^m a_{i1}^U\le\sum_{j=1}^n b_{j1}^L-\sum_{j=1}^n b_{j1}^U$, $\sum_{i=1}^m a_{i2}^U-\sum_{i=1}^m a_{i1}^L\le\sum_{j=1}^n b_{j2}^U-\sum_{j=1}^n b_{j1}^L$, $\sum_{i=1}^m a_{i2}^L-\sum_{i=1}^m a_{i2}^U\le\sum_{j=1}^n b_{j2}^L-\sum_{j=1}^n b_{j2}^U$, $\sum_{i=1}^m a_{i3}^L-\sum_{i=1}^m a_{i2}^L\le\sum_{j=1}^n b_{j3}^L-\sum_{j=1}^n b_{j2}^L$, $\sum_{i=1}^m a_{i3}^U-\sum_{i=1}^m a_{i3}^L\le\sum_{j=1}^n b_{j3}^U-\sum_{j=1}^n b_{j3}^L$, $\sum_{i=1}^m a_{i4}^L-\sum_{i=1}^m a_{i3}^U\le\sum_{j=1}^n b_{j4}^L-\sum_{j=1}^n b_{j3}^U$, $\sum_{i=1}^m a_{i4}^U-\sum_{i=1}^m a_{i4}^L\le\sum_{j=1}^n b_{j4}^U-\sum_{j=1}^n b_{j4}^L$ then add a dummy source S_{m+1} having dummy supply $\langle(A_{(m+1)1}^L,\ A_{(m+1)2}^L,\ A_{(m+1)3}^L,\ A_{(m+1)4}^L),\ (A_{(m+1)1}^U,\ A_{(m+1)2}^U,\ A_{(m+1)3}^U,\ A_{(m+1)4}^U)\rangle$ by considering the cost for supplying the unit quantity of the product from the dummy source S_{m+1} to all the destinations D_{n+1} as a generalized IVTrFN $\tilde{\tilde{0}}=\langle(0,0,0,0;1),(0,0,0,0;1)\rangle$.

$$A^U_{(m+1)1} = \sum_{j=1}^{n} b^U_{j1} - \sum_{i=1}^{m} a^U_{i1}, A^L_{(m+1)1} = A^U_{(m+1)1} + \left(\sum_{j=1}^{n} b^L_{j1} - \sum_{j=1}^{n} b^U_{j1}\right) - \left(\sum_{i=1}^{m} a^L_{i1} - \sum_{i=1}^{m} a^U_{i1}\right),$$

$$A^U_{(m+1)2} = A^L_{(m+1)1} + \left(\sum_{j=1}^{n} b^U_{j2} - \sum_{j=1}^{n} b^L_{j1}\right) - \left(\sum_{i=1}^{m} a^U_{i2} - \sum_{i=1}^{m} a^L_{i1}\right),$$

$$A^L_{(m+1)2} = A^U_{(m+1)2} + \left(\sum_{j=1}^{n} b^L_{j2} - \sum_{j=1}^{n} b^U_{j2}\right) - \left(\sum_{i=1}^{m} a^L_{i2} - \sum_{i=1}^{m} a^U_{i2}\right),$$

$$A^L_{(m+1)3} = A^L_{(m+1)2} + \left(\sum_{j=1}^{n} b^L_{j3} - \sum_{j=1}^{n} b^L_{j2}\right) - \left(\sum_{i=1}^{m} a^L_{i3} - \sum_{i=1}^{m} a^L_{i2}\right),$$

$$A^U_{(m+1)3} = A^L_{(m+1)3} + \left(\sum_{j=1}^{n} b^U_{j3} - \sum_{j=1}^{n} b^L_{j3}\right) - \left(\sum_{i=1}^{m} a^U_{i3} - \sum_{i=1}^{m} a^L_{i3}\right),$$

$$A^L_{(m+1)4} = A^U_{(m+1)3} + \left(\sum_{j=1}^{n} b^L_{j4} - \sum_{j=1}^{n} b^U_{j3}\right) - \left(\sum_{i=1}^{m} a^L_{i4} - \sum_{i=1}^{m} a^U_{i3}\right),$$

$$A^U_{(m+1)4} = A^L_{(m+1)4} + \left(\sum_{j=1}^{n} b^U_{j4} - \sum_{j=1}^{n} b^L_{j4}\right) - \left(\sum_{i=1}^{m} a^U_{i4} - \sum_{i=1}^{m} a^L_{i4}\right).$$

Case 2: If $\sum_{j=1}^{n} b^U_{j1} \le \sum_{i=1}^{m} a^U_{i1}$, $\sum_{j=1}^{n} b^L_{j1} - \sum_{j=1}^{n} b^U_{j1} \le \sum_{i=1}^{m} a^L_{i1} - \sum_{i=1}^{m} a^U_{i1}$,

$\sum_{j=1}^{n} b^U_{j2} - \sum_{j=1}^{n} b^L_{j1} \le \sum_{i=1}^{m} a^U_{i2} - \sum_{i=1}^{m} a^L_{i1}$, $\sum_{j=1}^{n} b^L_{j2} - \sum_{j=1}^{n} b^U_{j2} \le \sum_{i=1}^{m} a^L_{i2} - \sum_{i=1}^{m} a^U_{i2}$,

$\sum_{j=1}^{n} b^L_{j3} - \sum_{j=1}^{n} b^L_{j2} \le \sum_{i=1}^{m} a^L_{i3} - \sum_{i=1}^{m} a^L_{i2}$, $\sum_{j=1}^{n} b^U_{j3} - \sum_{j=1}^{n} b^L_{j3} \le \sum_{i=1}^{m} a^U_{i3} - \sum_{i=1}^{m} a^L_{i3}$,

$\sum_{j=1}^{n} b^L_{j4} - \sum_{j=1}^{n} b^U_{j3} \le \sum_{i=1}^{m} a^L_{i4} - \sum_{i=1}^{m} a^U_{i3}$, $\sum_{j=1}^{n} b^U_{j4} - \sum_{j=1}^{n} b^L_{j4} \le \sum_{i=1}^{m} a^U_{i4} - \sum_{i=1}^{m} a^L_{i4}$ then add a dummy destination D_{n+1} having dummy demand $\langle (B^L_{(n+1)1}, B^L_{(n+1)2}, B^L_{(n+1)3}, B^L_{(n+1)4}), (B^U_{(n+1)1}, B^U_{(n+1)2}, B^U_{(n+1)3}, B^U_{(n+1)4}) \rangle$ by considering the cost for supplying the unit quantity of the product from all the sources to the dummy destination D_{n+1} as a generalized IVTrFN $\tilde{\tilde{0}} = ((0,0,0,0;1),(0,0,0,0;1))$.

$$B^U_{(n+1)1} = \sum_{i=1}^{m} a^U_{i1} - \sum_{j=1}^{n} b^U_{j1}, B^L_{(n+1)1} = B^U_{(n+1)1} + \left(\sum_{i=1}^{m} a^L_{i1} - \sum_{i=1}^{m} a^U_{i1}\right) - \left(\sum_{j=1}^{n} b^L_{j1} - \sum_{j=1}^{n} b^U_{j1}\right),$$

$$B^U_{(n+1)2} = B^L_{(n+1)1} + \left(\sum_{i=1}^{m} a^U_{i2} - \sum_{i=1}^{m} a^L_{i1}\right) - \left(\sum_{j=1}^{n} b^U_{j2} - \sum_{j=1}^{n} b^L_{j1}\right),$$

$$B^L_{(n+1)2} = B^U_{(n+1)2} + \left(\sum_{i=1}^{m} a^L_{i2} - \sum_{i=1}^{m} a^U_{i2}\right) - \left(\sum_{j=1}^{n} b^L_{j2} - \sum_{j=1}^{n} b^U_{j2}\right),$$

$$B^L_{(n+1)3}=B^L_{(n+1)2}+\left(\sum_{i=1}^{m}a^L_{i3}-\sum_{i=1}^{m}a^L_{i2}\right)-\left(\sum_{j=1}^{n}b^L_{j3}-\sum_{j=1}^{n}b^L_{j2}\right),$$

$$B^U_{(n+1)3}=B^L_{(n+1)3}+\left(\sum_{i=1}^{m}a^U_{i3}-\sum_{i=1}^{m}a^L_{i3}\right)-\left(\sum_{j=1}^{n}b^U_{j3}-\sum_{j=1}^{n}b^L_{j3}\right),$$

$$B^L_{(n+1)4}=B^U_{(n+1)3}+\left(\sum_{i=1}^{m}a^L_{i4}-\sum_{i=1}^{m}a^U_{i3}\right)-\left(\sum_{j=1}^{n}b^L_{j4}-\sum_{j=1}^{n}b^U_{j3}\right),$$

$$B^U_{(n+1)4}=B^L_{(n+1)4}+\left(\sum_{i=1}^{m}a^U_{i4}-\sum_{i=1}^{m}a^L_{i4}\right)-\left(\sum_{j=1}^{n}b^U_{j4}-\sum_{j=1}^{n}b^L_{j4}\right).$$

Case 3: If neither Case 1 nor Case 2 is satisfied, then carry out the following:

(1) Add a dummy source S_{m+1} having dummy supply $\left\langle (A^L_{(m+1)1}, A^L_{(m+1)2}, A^L_{(m+1)3}, A^L_{(m+1)4}), (A^U_{(m+1)1}, A^U_{(m+1)2}, A^U_{(m+1)3}, A^U_{(m+1)4})\right\rangle$ by considering the cost for supplying the unit quantity of the product from the dummy source S_{m+1} to all the destinations as a generalized IVTrFN $\tilde{\tilde{0}}=\left((0,0,0,0;1),(0,0,0,0;1)\right)$.

$$A^U_{(m+1)1}=\max\left\{0,\left(\sum_{j=1}^{n}b^U_{j1}-\sum_{i=1}^{m}a^U_{i1}\right)\right\},$$

$$A^L_{(m+1)1}=A^U_{(m+1)1}+\max\left\{0,\left(\left(\sum_{j=1}^{n}b^L_{j1}-\sum_{j=1}^{n}b^U_{j1}\right)-\left(\sum_{i=1}^{m}a^L_{i1}-\sum_{i=1}^{m}a^U_{i1}\right)\right)\right\},$$

$$A^U_{(m+1)2}=A^L_{(m+1)1}+\max\left\{0,\left(\left(\sum_{j=1}^{n}b^U_{j2}-\sum_{j=1}^{n}b^L_{j1}\right)-\left(\sum_{i=1}^{m}a^U_{i2}-\sum_{i=1}^{m}a^L_{i1}\right)\right)\right\},$$

$$A^L_{(m+1)2}=A^U_{(m+1)2}+\max\left\{0,\left(\left(\sum_{j=1}^{n}b^L_{j2}-\sum_{j=1}^{n}b^U_{j2}\right)-\left(\sum_{i=1}^{m}a^L_{i2}-\sum_{i=1}^{m}a^U_{i2}\right)\right)\right\},$$

$$A^L_{(m+1)3}=A^L_{(m+1)2}+\max\left\{0,\left(\left(\sum_{j=1}^{n}b^L_{j3}-\sum_{j=1}^{n}b^L_{j2}\right)-\left(\sum_{i=1}^{m}a^L_{i3}-\sum_{i=1}^{m}a^L_{i2}\right)\right)\right\},$$

$$A^U_{(m+1)3}=A^L_{(m+1)3}+\max\left\{0,\left(\left(\sum_{j=1}^{n}b^U_{j3}-\sum_{j=1}^{n}b^L_{j3}\right)-\left(\sum_{i=1}^{m}a^U_{i3}-\sum_{i=1}^{m}a^L_{i3}\right)\right)\right\},$$

$$A^L_{(m+1)4}=A^U_{(m+1)3}+\max\left\{0,\left(\left(\sum_{j=1}^{n}b^L_{j4}-\sum_{j=1}^{n}b^U_{j3}\right)-\left(\sum_{i=1}^{m}a^L_{i4}-\sum_{i=1}^{m}a^U_{i3}\right)\right)\right\},$$

$$A^U_{(m+1)4}=A^L_{(m+1)4}+\max\left\{0,\left(\left(\sum_{j=1}^{n}b^U_{j4}-\sum_{j=1}^{n}b^L_{j4}\right)-\left(\sum_{i=1}^{m}a^U_{i4}-\sum_{i=1}^{m}a^L_{i4}\right)\right)\right\}$$

(2) Add a dummy destination D_{n+1} having the dummy demand $\left\langle (B^L_{(m+1)1}, B^L_{(m+1)2}, B^L_{(m+1)3}, B^L_{(m+1)4}), (B^U_{(m+1)1}, B^U_{(m+1)2}, B^U_{(m+1)3}, B^U_{(m+1)4})\right\rangle$ by considering the cost for supplying the unit quantity of the product from all the sources to the dummy destination D_{n+1} as a generalized IVTrFN $\tilde{\tilde{0}} = \left((0,0,0,0;1),(0,0,0,0;1)\right)$.

$$B^U_{(n+1)1} = \max\left\{0, \left(\sum_{i=1}^{m} a^U_{i1} - \sum_{j=1}^{n} b^U_{j1}\right)\right\},$$

$$B^L_{(n+1)1} = B^U_{(n+1)1} + \max\left\{0, \left(\left(\sum_{i=1}^{m} a^L_{i1} - \sum_{i=1}^{m} a^U_{i1}\right) - \left(\sum_{j=1}^{n} b^L_{j1} - \sum_{j=1}^{n} b^U_{j1}\right)\right)\right\},$$

$$B^U_{(n+1)2} = B^L_{(n+1)1} + \max\left\{0, \left(\left(\sum_{i=1}^{m} a^U_{i2} - \sum_{i=1}^{m} a^L_{i1}\right) - \left(\sum_{j=1}^{n} b^U_{j2} - \sum_{j=1}^{n} b^L_{j1}\right)\right)\right\},$$

$$B^L_{(n+1)2} = B^U_{(n+1)2} + \max\left\{0, \left(\left(\sum_{i=1}^{m} a^L_{i2} - \sum_{i=1}^{m} a^U_{i2}\right) - \left(\sum_{j=1}^{n} b^L_{j2} - \sum_{j=1}^{n} b^U_{j2}\right)\right)\right\},$$

$$B^L_{(n+1)3} = B^L_{(n+1)2} + \max\left\{0, \left(\left(\sum_{i=1}^{m} a^L_{i3} - \sum_{i=1}^{m} a^L_{i2}\right) - \left(\sum_{j=1}^{n} b^L_{j3} - \sum_{j=1}^{n} b^L_{j2}\right)\right)\right\},$$

$$B^U_{(n+1)3} = B^L_{(n+1)3} + \max\left\{0, \left(\left(\sum_{i=1}^{m} a^U_{i3} - \sum_{i=1}^{m} a^L_{i3}\right) - \left(\sum_{j=1}^{n} b^U_{j3} - \sum_{j=1}^{n} b^L_{j3}\right)\right)\right\},$$

$$B^L_{(n+1)4} = B^U_{(n+1)3} + \max\left\{0, \left(\left(\sum_{i=1}^{m} a^L_{i4} - \sum_{i=1}^{m} a^U_{i3}\right) - \left(\sum_{j=1}^{n} b^L_{j4} - \sum_{j=1}^{n} b^U_{j3}\right)\right)\right\},$$

$$B^U_{(n+1)4} = B^L_{(n+1)4} + \max\left\{0, \left(\left(\sum_{i=1}^{m} a^U_{i4} - \sum_{i=1}^{m} a^L_{i4}\right) - \left(\sum_{j=1}^{n} b^U_{j4} - \sum_{j=1}^{n} b^L_{j4}\right)\right)\right\}.$$

5 Validity of the proposed Mehar method

To prove the validity of the proposed Mehar method, it is sufficient to prove that:

1. The obtained dummy supply will be a generalized IVTrFN.
2. The obtained dummy demand will be a generalized IVTrFN.
3. In Case 1, $\sum_{i=1}^{m} \tilde{\tilde{a}}_i + \tilde{\tilde{A}}_{m+1} = \sum_{j=1}^{n} \tilde{\tilde{b}}_j$.

4. In Case 2, $\sum_{i=1}^{m}\tilde{\tilde{a}}_i = \sum_{j=1}^{n}\tilde{\tilde{b}}_j + \tilde{\tilde{B}}_{n+1}$.

5. In Case 3, $\sum_{i=1}^{m}\tilde{\tilde{a}}_i + \tilde{\tilde{A}}_{m+1} = \sum_{j=1}^{n}\tilde{\tilde{b}}_j + \tilde{\tilde{B}}_{n+1}$.

Therefore, the same is proved in this section.

5.1 The obtained dummy supply will be a generalized IVTrFN

It is obvious from Case 1 of the proposed Mehar method that for the obtained dummy supply $\left\langle (A^L_{(m+1)1}, A^L_{(m+1)2}, A^L_{(m+1)3}, A^L_{(m+1)4}), (A^U_{(m+1)1}, A^U_{(m+1)2}, A^U_{(m+1)3}, A^U_{(m+1)4}) \right\rangle$, the conditions $A^U_{(m+1)1} \le A^L_{(m+1)1} \le A^U_{(m+1)2} \le A^L_{(m+1)2} \le A^L_{(m+1)3} \le A^U_{(m+1)3} \le A^L_{(m+1)4} \le A^U_{(m+1)4}$, will be satisfied. Therefore, the dummy supply $\left\langle (A^L_{(m+1)1}, A^L_{(m+1)2}, A^L_{(m+1)3}, A^L_{(m+1)4}), (A^U_{(m+1)1}, A^U_{(m+1)2}, A^U_{(m+1)3}, A^U_{(m+1)4}) \right\rangle$, obtained from Case 1 of the proposed Mehar method, will always be a generalized IVTrFN.

5.2 The obtained dummy demand will be a generalized IVTrFN

It is obvious from Case 2 of the proposed Mehar method that for the obtained dummy demand $\left\langle (B^L_{(n+1)1}, B^L_{(n+1)2}, B^L_{(n+1)3}, B^L_{(n+1)4}), (B^U_{(n+1)1}, B^U_{(n+1)2}, B^U_{(n+1)3}, B^U_{(n+1)4}) \right\rangle$, the conditions $B^U_{(n+1)1} \le B^L_{(n+1)1} \le B^U_{(n+1)2} \le B^L_{(n+1)2} \le B^U_{(n+1)3} \le B^L_{(n+1)3} \le B^U_{(n+1)4} \le B^L_{(n+1)4}$, will be satisfied. Therefore, the dummy demand $\left\langle (B^L_{(n+1)1}, B^L_{(n+1)2}, B^L_{(n+1)3}, B^L_{(n+1)4}), (B^U_{(n+1)1}, B^U_{(n+1)2}, B^U_{(n+1)3}, B^U_{(n+1)4}) \right\rangle$, obtained from the Case 2 of the proposed Mehar method, will always be a generalized IVTrFN.

5.3 The obtained dummy supply and dummy demand will be a generalized IVTrFNs

It is obvious from Case 3 of the proposed Mehar method that for the obtained dummy supply $\left\langle (A^L_{(m+1)1}, A^L_{(m+1)2}, A^L_{(m+1)3}, A^L_{(m+1)4}), (A^U_{(m+1)1}, A^U_{(m+1)2}, A^U_{(m+1)3}, A^U_{(m+1)4}) \right\rangle$, the conditions $A^U_{(m+1)1} \le A^L_{(m+1)1} \le A^U_{(m+1)2} \le A^L_{(m+1)2} \le A^L_{(m+1)3} \le A^U_{(m+1)3} \le A^L_{(m+1)4} \le A^U_{(m+1)4}$, will be satisfied. In addition, for the obtained dummy demand $\left\langle (B^L_{(n+1)1}, B^L_{(n+1)2}, B^L_{(n+1)3}, B^L_{(n+1)4}), (B^U_{(n+1)1}, B^U_{(n+1)2}, B^U_{(n+1)3}, B^U_{(n+1)4}) \right\rangle$, the conditions $B^U_{(n+1)1} \le B^L_{(n+1)1} \le B^U_{(n+1)2} \le B^L_{(n+1)2} \le B^L_{(n+1)3} \le B^U_{(n+1)3} \le B^L_{(n+1)4} \le B^U_{(n+1)4}$, will be satisfied. Therefore, the dummy supply $\left\langle (A^L_{(m+1)1}, A^L_{(m+1)2}, A^L_{(m+1)3}, A^L_{(m+1)4}), (A^U_{(m+1)1}, A^U_{(m+1)2}, A^U_{(m+1)3}, A^U_{(m+1)4}) \right\rangle$ and the dummy demand $\left\langle (B^L_{(n+1)1}, B^L_{(n+1)2}, B^L_{(n+1)3}, B^L_{(n+1)4}), (B^U_{(n+1)1}, B^U_{(n+1)2}, B^U_{(n+1)3}, B^U_{(n+1)4}) \right\rangle$, obtained from Case 3 of the proposed Mehar method, will always be a generalized IVTrFNs.

5.4 Validity of the condition $\sum_{i=1}^{m}\tilde{\tilde{a}}_i + \tilde{\tilde{A}}_{m+1} = \sum_{j=1}^{n}\tilde{\tilde{b}}_j$

$$\sum_{i=1}^{m}\tilde{\tilde{a}}_i + \tilde{\tilde{A}}_{m+1} = \sum_{j=1}^{n}\tilde{\tilde{b}}_j$$

$$\Rightarrow \left\langle \left(\sum_{i=1}^{m} a_{i1}^L, \sum_{i=1}^{m} a_{i2}^L, \sum_{i=1}^{m} a_{i3}^L, \sum_{i=1}^{m} a_{i4}^L; \omega^L \right), \left(\sum_{i=1}^{m} a_{i1}^U, \sum_{i=1}^{m} a_{i2}^U, \sum_{i=1}^{m} a_{i3}^U, \sum_{i=1}^{m} a_{i4}^U; \omega^U \right) \right\rangle$$
$$+ \left\langle \left(A_{(m+1)1}^L, A_{(m+1)2}^L, A_{(m+1)3}^L, A_{(m+1)4}^L \right), \left(A_{(m+1)1}^U, A_{(m+1)2}^U, A_{(m+1)3}^U, A_{(m+1)4}^U \right) \right\rangle$$
$$= \left\langle \left(\sum_{j=1}^{n} b_{j1}^L, \sum_{j=1}^{n} b_{j2}^L, \sum_{j=1}^{n} b_{j3}^L, \sum_{j=1}^{n} b_{j4}^L; \omega^L \right), \left(\sum_{j=1}^{n} b_{j1}^U, \sum_{j=1}^{n} b_{j2}^U, \sum_{j=1}^{n} b_{j3}^U, \sum_{j=1}^{n} b_{j4}^U; \omega^U \right) \right\rangle.$$

$$\Rightarrow \left\langle \begin{array}{l} \left(\sum_{i=1}^{m} a_{i1}^L + A_{(m+1)1}^L, \sum_{i=1}^{m} a_{i2}^L + A_{(m+1)2}^L, \sum_{i=1}^{m} a_{i3}^L + A_{(m+1)3}^L, \sum_{i=1}^{m} a_{i4}^L + A_{(m+1)4}^L; \omega^L \right), \\ \left(\sum_{i=1}^{m} a_{i1}^U + A_{(m+1)1}^U, \sum_{i=1}^{m} a_{i2}^U + A_{(m+1)2}^U, \sum_{i=1}^{m} a_{i3}^U + A_{(m+1)3}^U, \sum_{i=1}^{m} a_{i4}^U + A_{(m+1)}^U; \omega^U \right) \end{array} \right\rangle$$
$$= \left\langle \left(\sum_{j=1}^{n} b_{j1}^L, \sum_{j=1}^{n} b_{j2}^L, \sum_{j=1}^{n} b_{j3}^L, \sum_{j=1}^{n} b_{j4}^L; \omega^L \right), \left(\sum_{j=1}^{n} b_{j1}^U, \sum_{j=1}^{n} b_{j2}^U, \sum_{j=1}^{n} b_{j3}^U, \sum_{j=1}^{n} b_{j4}^U; \omega^U \right) \right\rangle.$$

$$\Rightarrow \sum_{i=1}^{m} a_{i1}^L + A_{(m+1)1}^L = \sum_{j=1}^{n} b_{j1}^L, \tag{1}$$

$$\sum_{i=1}^{m} a_{i2}^L + A_{(m+1)2}^L = \sum_{j=1}^{n} b_{j2}^L, \tag{2}$$

$$\sum_{i=1}^{m} a_{i3}^L + A_{(m+1)3}^L = \sum_{j=1}^{n} b_{j3}^L, \tag{3}$$

$$\sum_{i=1}^{m} a_{i4}^L + A_{(m+1)4}^L = \sum_{j=1}^{n} b_{j4}^L, \tag{4}$$

$$\sum_{i=1}^{m} a_{i1}^U + A_{(m+1)1}^U = \sum_{j=1}^{n} b_{j1}^U, \tag{5}$$

$$\sum_{i=1}^{m} a_{i2}^U + A_{(m+1)2}^U = \sum_{j=1}^{n} b_{j2}^U, \tag{6}$$

$$\sum_{i=1}^{m} a_{i3}^U + A_{(m+1)3}^U = \sum_{j=1}^{n} b_{j3}^U, \tag{7}$$

$$\sum_{i=1}^{m} a_{i4}^{U} + A_{(m+1)4}^{U} = \sum_{j=1}^{n} b_{j4}^{U}. \tag{8}$$

It is obvious that in order to prove $\sum_{i=1}^{m} \tilde{\tilde{a}}_i + \tilde{\tilde{A}}_{m+1} = \sum_{j=1}^{n} \tilde{\tilde{b}}_j$, there is a need to prove that Eqs. (1)–(8) are satisfied. Here, only the validity of Eq. (1) is proved. The validity of the remaining equations can be proved in the same manner.

$$\begin{aligned}\sum_{i=1}^{m} a_{i1}^{L} + A_{(m+1)1}^{L} &= \sum_{i=1}^{m} a_{i1}^{L} + A_{(m+1)1}^{U} + \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) - \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)\\ &= \sum_{i=1}^{m} a_{i1}^{L} + \sum_{j=1}^{n} b_{j1}^{U} - \sum_{i=1}^{m} a_{i1}^{U} + \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) - \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)\\ &= \sum_{j=1}^{n} b_{j1.}^{L}\end{aligned}$$

5.5 Validity of the condition $\sum_{i=1}^{m} \tilde{\tilde{a}}_i = \sum_{j=1}^{n} \tilde{\tilde{b}}_j + \tilde{\tilde{B}}_{n+1}$

$$\sum_{i=1}^{m} \tilde{\tilde{a}}_i = \sum_{j=1}^{n} \tilde{\tilde{b}}_j + \tilde{\tilde{B}}_{n+1}$$

$$\begin{aligned}&\Rightarrow \left\langle \left(\sum_{i=1}^{m} a_{i1}^{L}, \sum_{i=1}^{m} a_{i2}^{L}, \sum_{i=1}^{m} a_{i3}^{L}, \sum_{i=1}^{m} a_{i4}^{L}; \omega^{L}\right), \left(\sum_{i=1}^{m} a_{i1}^{U}, \sum_{i=1}^{m} a_{i2}^{U}, \sum_{i=1}^{m} a_{i3}^{U}, \sum_{i=1}^{m} a_{i4}^{U}; \omega^{U}\right)\right\rangle\\ &= \left\langle \left(\sum_{j=1}^{n} b_{j1}^{L}, \sum_{j=1}^{n} b_{j2}^{L}, \sum_{j=1}^{n} b_{j3}^{L}, \sum_{j=1}^{n} b_{j4}^{L}; \omega^{L}\right), \left(\sum_{j=1}^{n} b_{j1}^{U}, \sum_{j=1}^{n} b_{j2}^{U}, \sum_{j=1}^{n} b_{j3}^{U}, \sum_{j=1}^{n} b_{j4}^{U}; \omega^{U}\right)\right\rangle\\ &+ \left\langle \left(B_{(n+1)1}^{L}, B_{(n+1)2}^{L}, B_{(n+1)3}^{L}, B_{(n+1)4}^{L}\right), \left(B_{(n+1)1}^{U}, B_{(n+1)2}^{U}, B_{(n+1)3}^{U}, B_{(n+1)4}^{U}\right)\right\rangle.\end{aligned}$$

$$\begin{aligned}&\Rightarrow \left\langle \left(\sum_{i=1}^{m} a_{i1}^{L}, \sum_{i=1}^{m} a_{i2}^{L}, \sum_{i=1}^{m} a_{i3}^{L}, \sum_{i=1}^{m} a_{i4}^{L}; \omega^{L}\right), \left(\sum_{i=1}^{m} a_{i1}^{U}, \sum_{i=1}^{m} a_{i2}^{U}, \sum_{i=1}^{m} a_{i3}^{U}, \sum_{i=1}^{m} a_{i4}^{U}; \omega^{U}\right)\right\rangle\\ &= \left\langle \begin{array}{l}\left(\sum_{j=1}^{n} b_{j1}^{L} + B_{(n+1)1}^{L}, \sum_{j=1}^{n} b_{j2}^{L} + B_{(n+1)2}^{L}, \sum_{j=1}^{n} b_{j3}^{L} + B_{(n+1)3}^{L}, \sum_{j=1}^{n} b_{j4}^{L} + B_{(n+1)4}^{L}; \omega^{L}\right),\\ \left(\sum_{j=1}^{n} b_{j1}^{U} + B_{(n+1)1}^{U}, \sum_{j=1}^{n} b_{j2}^{U} + B_{(n+1)2}^{U}, \sum_{j=1}^{n} b_{j3}^{U} + B_{(n+1)3}^{U}, \sum_{j=1}^{n} b_{j4}^{U} + B_{(n+1)4}^{U}; \omega^{U}\right)\end{array}\right\rangle.\end{aligned}$$

$$\Rightarrow \sum_{i=1}^{m} a_{i1}^{L} = \sum_{j=1}^{n} b_{j1}^{L} + B_{(n+1)1}^{L}, \tag{9}$$

$$\sum_{i=1}^{m} a_{i2}^{L} = \sum_{j=1}^{n} b_{j2}^{L} + B_{(n+1)2}^{L}, \tag{10}$$

$$\sum_{i=1}^{m} a_{i3}^{L} = \sum_{j=1}^{n} b_{j3}^{L} + B_{(n+1)3}^{L}, \tag{11}$$

$$\sum_{i=1}^{m} a_{i4}^{L} = \sum_{j=1}^{n} b_{j4}^{L} + B_{(n+1)4}^{L}, \tag{12}$$

$$\sum_{i=1}^{m} a_{i1}^{U} = \sum_{j=1}^{n} b_{j1}^{U} + B_{(n+1)1}^{U}, \tag{13}$$

$$\sum_{i=1}^{m} a_{i2}^{U} = \sum_{j=1}^{n} b_{j2}^{U} + B_{(n+1)2}^{U}, \tag{14}$$

$$\sum_{i=1}^{m} a_{i3}^{U} = \sum_{j=1}^{n} b_{j3}^{U} + B_{(n+1)3}^{U}, \tag{15}$$

$$\sum_{i=1}^{m} a_{i4}^{U} = \sum_{j=1}^{n} b_{j4}^{U} + B_{(n+1)4}^{U}. \tag{16}$$

It is obvious that in order to prove $\sum_{i=1}^{m} \tilde{\tilde{a}}_i = \sum_{j=1}^{n} \tilde{\tilde{b}}_j + \tilde{\tilde{B}}_{n+1}$, there is a need to prove that Eqs. (9)–(16) are satisfied. Here, only the validity of Eq. (9) is proved. The validity of the remaining equations can be proved in the same manner.

$$\begin{aligned}
\sum_{j=1}^{n} b_{j1}^{L} + B_{(n+1)1}^{L} &= \sum_{j=1}^{n} b_{j1}^{L} + B_{(n+1)1}^{U} + \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right) - \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) \\
&= \sum_{j=1}^{n} b_{j1}^{L} + \sum_{i=1}^{m} a_{i1}^{U} - \sum_{j=1}^{n} b_{j1}^{U} + \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right) - \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) = \sum_{i=1}^{m} a_{i1}^{L}.
\end{aligned}$$

5.6 Validity of the condition $\sum_{i=1}^{m} \tilde{\tilde{a}}_i + \tilde{\tilde{A}}_{m+1} = \sum_{j=1}^{n} \tilde{\tilde{b}}_j + \tilde{\tilde{B}}_{n+1}$

$$\sum_{i=1}^{m} \tilde{\tilde{a}}_i + \tilde{\tilde{A}}_{m+1} = \sum_{j=1}^{n} \tilde{\tilde{b}}_j + \tilde{\tilde{B}}_{n+1}.$$

$$\begin{aligned}
\Rightarrow &\left\langle \left(\left(\sum_{i=1}^{m} a_{i1}^{L}, \sum_{i=1}^{m} a_{i2}^{L}, \sum_{i=1}^{m} a_{i3}^{L}, \sum_{i=1}^{m} a_{i4}^{L};\ \omega^{L} \right), \left(\sum_{i=1}^{m} a_{i1}^{U}, \sum_{i=1}^{m} a_{i2}^{U}, \sum_{i=1}^{m} a_{i3}^{U}, \sum_{i=1}^{m} a_{i4}^{U};\ \omega^{U} \right) \right) \right\rangle \\
&+ \left\langle \left(\left(A_{(m+1)1}^{L}, A_{(m+1)2}^{L}, A_{(m+1)3}^{L}, A_{(m+1)4}^{L} \right), \left(A_{(m+1)1}^{U}, A_{(m+1)2}^{U}, A_{(m+1)3}^{U}, A_{(m+1)4}^{U} \right) \right) \right\rangle \\
&= \left\langle \left(\sum_{j=1}^{n} b_{j1}^{L}, \sum_{j=1}^{n} b_{j2}^{L}, \sum_{j=1}^{n} b_{j3}^{L}, \sum_{j=1}^{n} b_{j4}^{L};\ \omega^{L} \right), \left(\sum_{j=1}^{n} b_{j1}^{U}, \sum_{j=1}^{n} b_{j2}^{U}, \sum_{j=1}^{n} b_{j3}^{U}, \sum_{j=1}^{n} b_{j4}^{U};\ \omega^{U} \right) \right\rangle \\
&+ \left\langle \left(B_{(n+1)1}^{L}, B_{(n+1)2}^{L}, B_{(n+1)3}^{L}, B_{(n+1)4}^{L} \right), \left(B_{(n+1)1}^{U}, B_{(n+1)2}^{U}, B_{(n+1)3}^{U}, B_{(n+1)4}^{U} \right) \right\rangle.
\end{aligned}$$

$$\Rightarrow \left\langle \begin{array}{l} \left(\sum_{i=1}^{m} a_{i1}^{L} + A_{(m+1)1}^{L}, \sum_{i=1}^{m} a_{i2}^{L} + A_{(m+1)2}^{L}, \sum_{i=1}^{m} a_{i3}^{L} + A_{(m+1)3}^{L}, \sum_{i=1}^{m} a_{i4}^{L} + A_{(m+1)4}^{L}; \omega^{L} \right), \\ \left(\sum_{i=1}^{m} a_{i1}^{U} + A_{(m+1)1}^{U}, \sum_{i=1}^{m} a_{i2}^{U} + A_{(m+1)2}^{U}, \sum_{i=1}^{m} a_{i3}^{U} + A_{(m+1)3}^{U}, \sum_{i=1}^{m} a_{i4}^{U} + A_{(m+1)}^{U}; \omega^{U} \right) \end{array} \right\rangle$$

$$= \left\langle \begin{array}{l} \left(\sum_{j=1}^{n} b_{j1}^{L} + B_{(n+1)1}^{L}, \sum_{j=1}^{n} b_{j2}^{L} + B_{(n+1)2}^{L}, \sum_{j=1}^{n} b_{j3}^{L} + B_{(n+1)3}^{L}, \sum_{j=1}^{n} b_{j4}^{L} + B_{(n+1)4}^{L}; \omega^{L} \right), \\ \left(\sum_{j=1}^{n} b_{j1}^{U} + B_{(n+1)1}^{U}, \sum_{j=1}^{n} b_{j2}^{U} + B_{(n+1)2}^{U}, \sum_{j=1}^{n} b_{j3}^{U} + B_{(n+1)3}^{U}, \sum_{j=1}^{n} b_{j4}^{U} + B_{(n+1)4}^{U}; \omega^{U} \right) \end{array} \right\rangle.$$

$$\Rightarrow \sum_{i=1}^{m} a_{i1}^{L} + A_{(m+1)1}^{L} = \sum_{j=1}^{n} b_{j1}^{L} + B_{(n+1)1}^{L}, \tag{17}$$

$$\sum_{i=1}^{m} a_{i2}^{L} + A_{(m+1)2}^{L} = \sum_{j=1}^{n} b_{j2}^{L} + B_{(n+1)2}^{L}, \tag{18}$$

$$\sum_{i=1}^{m} a_{i3}^{L} + A_{(m+1)3}^{L} = \sum_{j=1}^{n} b_{j3}^{L} + B_{(n+1)3}^{L}, \tag{19}$$

$$\sum_{i=1}^{m} a_{i4}^{L} + A_{(m+1)4}^{L} = \sum_{j=1}^{n} b_{j4}^{L} + B_{(n+1)4}^{L}, \tag{20}$$

$$\sum_{i=1}^{m} a_{i1}^{U} + A_{(m+1)1}^{U} = \sum_{j=1}^{n} b_{j1}^{U} + B_{(n+1)1}^{U}, \tag{21}$$

$$\sum_{i=1}^{m} a_{i2}^{U} + A_{(m+1)2}^{U} = \sum_{j=1}^{n} b_{j2}^{U} + B_{(n+1)2}^{U}, \tag{22}$$

$$\sum_{i=1}^{m} a_{i3}^{U} + A_{(m+1)3}^{U} = \sum_{j=1}^{n} b_{j3}^{U} + B_{(n+1)3}^{U}, \tag{23}$$

$$\sum_{i=1}^{m} a_{i4}^{U} + A_{(m+1)4}^{U} = \sum_{j=1}^{n} b_{j4}^{U} + B_{(n+1)4}^{U}. \tag{24}$$

It is obvious that in order to prove $\sum_{i=1}^{m} \tilde{\tilde{a}}_i + \tilde{\tilde{A}}_{m+1} = \sum_{j=1}^{n} \tilde{\tilde{b}}_j + \tilde{\tilde{B}}_{n+1}$, there is a need to prove that Eqs. (17)–(24) are satisfied. Here, only the validity of Eq. (17) is proved. The validity of the remaining equations can be proved in the same manner.

Putting the values of $A_{(m+1)1}^{L}$ and $B_{(n+1)1}^{L}$ in Eq. (17), it will be transformed into Eq. (25).

$$\begin{aligned} &\sum_{i=1}^{m} a_{i1}^{L} + \max\left\{0, \left(\sum_{j=1}^{n} b_{j1}^{U} - \sum_{i=1}^{m} a_{i1}^{U}\right)\right\} + \max\left\{0, \left(\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) - \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)\right)\right\} \\ &= \sum_{j=1}^{n} b_{j1}^{L} + \max\left\{0, \left(\sum_{i=1}^{m} a_{i1}^{U} - \sum_{j=1}^{n} b_{j1}^{U}\right)\right\} + \max\left\{0, \left(\left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right) - \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right)\right)\right\} \end{aligned} \tag{25}$$

There may be the following four cases:

Case 1: $\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) \geq \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)$ and $\sum_{j=1}^{n} b_{j1}^{U} \geq \sum_{i=1}^{m} a_{i1}^{U}$.

Case 2: $\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) \leq \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)$ and $\sum_{j=1}^{n} b_{j1}^{U} \leq \sum_{i=1}^{m} a_{i1}^{U}$.

Case 3: $\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) \leq \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)$ and $\sum_{j=1}^{n} b_{j1}^{U} \geq \sum_{i=1}^{m} a_{i1}^{U}$.

Case 4: $\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) \geq \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)$ and $\sum_{j=1}^{n} b_{j1}^{U} \leq \sum_{i=1}^{m} a_{i1}^{U}$.

If $\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) \geq \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)$ and $\sum_{j=1}^{n} b_{j1}^{U} \geq \sum_{i=1}^{m} a_{i1}^{U}$ then

$$\max\left\{0, \left(\left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right) - \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right)\right)\right\} = 0, \quad \max\left\{0, \left(\sum_{i=1}^{m} a_{i1}^{U} - \sum_{j=1}^{n} b_{j1}^{U}\right)\right\} = 0,$$

$$\max\left\{0, \left(\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) - \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)\right)\right\}, \quad \max\left\{0, \left(\sum_{j=1}^{n} b_{j1}^{U} - \sum_{i=1}^{m} a_{i1}^{U}\right)\right\} = \sum_{j=1}^{n} b_{j1}^{U} - \sum_{i=1}^{m} a_{i1}^{U}$$

$$= \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) - \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)$$

and therefore Eq. (25) will be satisfied.

If $\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) \leq \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)$ and $\sum_{j=1}^{n} b_{j1}^{U} \leq \sum_{i=1}^{m} a_{i1}^{U}$ then

$$\max\left\{0, \left(\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) - \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)\right)\right\} = 0, \quad \max\left\{0, \left(\sum_{j=1}^{n} b_{j1}^{U} - \sum_{i=1}^{m} a_{i1}^{U}\right)\right\} = 0,$$

$$\max\left\{0, \left(\left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right) - \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right)\right)\right\} = \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right) - \left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right),$$

$$\max\left\{0, \left(\sum_{i=1}^{m} a_{i1}^{U} - \sum_{j=1}^{n} b_{j1}^{U}\right)\right\} = \sum_{i=1}^{m} a_{i1}^{U} - \sum_{j=1}^{n} b_{j1}^{U}$$

and therefore Eq. (25) will be satisfied.

If $\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) \leq \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)$ and $\sum_{j=1}^{n} b_{j1}^{U} \geq \sum_{i=1}^{m} a_{i1}^{U}$ then

$$\max\left\{0, \left(\left(\sum_{j=1}^{n} b_{j1}^{L} - \sum_{j=1}^{n} b_{j1}^{U}\right) - \left(\sum_{i=1}^{m} a_{i1}^{L} - \sum_{i=1}^{m} a_{i1}^{U}\right)\right)\right\} = 0, \quad \max\left\{0, \left(\sum_{j=1}^{n} b_{j1}^{U} - \sum_{i=1}^{m} a_{i1}^{U}\right)\right\} = 0,$$

$$\max\left\{0,\left(\left(\sum_{i=1}^{m}a_{i1}^{L}-\sum_{i=1}^{m}a_{i1}^{U}\right)-\left(\sum_{j=1}^{n}b_{j1}^{L}-\sum_{j=1}^{n}b_{j1}^{U}\right)\right)\right\}=\left(\sum_{i=1}^{m}a_{i1}^{L}-\sum_{i=1}^{m}a_{i1}^{U}\right)-\left(\sum_{j=1}^{n}b_{j1}^{L}-\sum_{j=1}^{n}b_{j1}^{U}\right),$$

$\max\left\{0,\left(\sum_{i=1}^{m}a_{i1}^{U}-\sum_{j=1}^{n}b_{j1}^{U}\right)\right\}=\sum_{i=1}^{m}a_{i1}^{U}-\sum_{j=1}^{n}b_{j1}^{U}$ and therefore Eq. (25) will be satisfied.

If $\left(\sum_{j=1}^{n}b_{j1}^{L}-\sum_{j=1}^{n}b_{j1}^{U}\right)\geq\left(\sum_{i=1}^{m}a_{i1}^{L}-\sum_{i=1}^{m}a_{i1}^{U}\right)$ and $\sum_{j=1}^{n}b_{j1}^{U}\leq\sum_{i=1}^{m}a_{i1}^{U}$ then the values of

$$\max\left\{0,\left(\left(\sum_{j=1}^{n}b_{j1}^{L}-\sum_{j=1}^{n}b_{j1}^{U}\right)-\left(\sum_{i=1}^{m}a_{i1}^{L}-\sum_{i=1}^{m}a_{i1}^{U}\right)\right)\right\}=0,\qquad \max\left\{0,\left(\sum_{j=1}^{n}b_{j1}^{U}-\sum_{i=1}^{m}a_{i1}^{U}\right)\right\}=0,$$

$$\max\left\{0,\left(\left(\sum_{i=1}^{m}a_{i1}^{L}-\sum_{i=1}^{m}a_{i1}^{U}\right)-\left(\sum_{j=1}^{n}b_{j1}^{L}-\sum_{j=1}^{n}b_{j1}^{U}\right)\right)\right\}=\left(\sum_{i=1}^{m}a_{i1}^{L}-\sum_{i=1}^{m}a_{i1}^{U}\right)-\left(\sum_{j=1}^{n}b_{j1}^{L}-\sum_{j=1}^{n}b_{j1}^{U}\right),$$

$\max\left\{0,\left(\sum_{i=1}^{m}a_{i1}^{U}-\sum_{j=1}^{n}b_{j1}^{U}\right)\right\}=\sum_{i=1}^{m}a_{i1}^{U}-\sum_{j=1}^{n}b_{j1}^{U}$ and therefore Eq. (25) will be satisfied.

6 Invalidity of the existing result

Ebrahimnejad (2016) considered a generalized IVTrFNTP having two sources, S_1, S_2, and three destinations, D_1, D_2, D_3, such that:

(i) The generalized IVTrF supplies at sources S_1 and S_2 are $\left\langle\left(70,90,90,100;\ \frac{2}{3}\right),(65,85,95,105;\ 1)\right\rangle$, $\left\langle\left(40,60,70,80;\ \frac{2}{3}\right),(35,55,75,85;\ 1)\right\rangle$, respectively.

(ii) The generalized IVTrF demands at destinations D_1, D_2 and D_3 are $\left\langle\left(30,40,50,70;\ \frac{2}{3}\right),(25,35,55,75;\ 1)\right\rangle$, $\left(\left(20,30,40,50;\frac{2}{3}\right),(15,25,45,55;1)\right)$ and $\left\langle\left(40,50,50,80;\ \frac{2}{3}\right),(35,45,55,85;\ 1)\right\rangle$, respectively.

Ebrahimnejad (2016) claimed that

$\left\langle\left(70,90,90,100;\ \frac{2}{3}\right),(65,85,95,105;\ 1)\right\rangle+\left\langle\left(40,60,70,80;\ \frac{2}{3}\right),(35,55,75,85;\ 1)\right\rangle$ is not equal to

$$=\left\langle\left(110,150,160,180;\ \frac{2}{3}\right),(35,55,75,85;\ 1)\right\rangle$$

$$\left\langle\left(30,40,50,70;\ \frac{2}{3}\right),(25,35,55,75,1)\right\rangle+\left\langle\left(20,30,40,50,\ \frac{2}{3}\right),(15,25,45,55;\ 1)\right\rangle$$

$$+\left\langle\left(40,50,50,80;\ \frac{2}{3}\right),(35,45,55,85;\ 1)\right\rangle=\left\langle\left(190,120,140,200;\ \frac{2}{3}\right),(75,105,155,215;\ 1)\right\rangle,$$

i.e., the considered generalized IVTrFNTP is an unbalanced generalized IVTrFNTP. Therefore, there is a need to add a dummy source S_3 having a dummy generalized IVTrF supply $\left\langle\left(25,25,35,75;\ \frac{2}{3}\right),\left(0,25,45,85;\ 1\right)\right\rangle$ and a dummy destination D_4 having a dummy generalized IVTrF demand $\left\langle\left(45,55,55,55;\ \frac{2}{3}\right),\left(25,60,60,60;\ 1\right)\right\rangle$ in order to solve the considered generalized IVTrFNTP.

However, the dummy generalized IVTrF demand $\left\langle\left(45,55,55,55;\ \frac{2}{3}\right),\left(25,60,60,60;\ 1\right)\right\rangle$, obtained by Ebrahimnejad (2016), is not a generalized IVTrFN for the following reason. It can be easily verified from the graphical representation of generalized IVTrFN (Ebrahimnejad, 2016) as well as from the existing definition of a generalized IVTrFN (Ebrahimnejad, 2016) that in a generalized IVTrFN $\langle(b_1^L, b_2^L, b_3^L, b_4^L; \omega^L), (b_1^U, b_2^U, b_3^U, b_4^U; \omega^U)\rangle$, the condition $b_1^U \le b_1^L \le b_2^U \le {b_2}^L \le b_3^L \le b_3^U \le b_4^L \le b_4^U$ should always be satisfied. While it can be easily verified that if the generalized IVTrF demand $\left\langle\left(45,55,55,55;\ \frac{2}{3}\right),\left(25,60,60,60;\ 1\right)\right\rangle$ is compared with a generalized IVTrFN $\langle(b_1^L, b_2^L, b_3^L, b_4^L; \omega^L), (b_1^U, b_2^U, b_3^U, b_4^U; \omega^U)\rangle$, then $b_1^L = 45$, $b_2^L = 55$, $b_3^L = 55$, $b_4^L = 55$, $b_1^U = 25$, $b_2^U = 60$, $b_3^U = 60$, ${b_4}^U = 60$.

It is obvious that $b_2^U \ge b_2^L$, i.e., the necessary condition $b_2^U \le b_2^L$ is not satisfied. Therefore, the obtained dummy demand is not a generalized IVTrFN. Hence, the result of this problem, obtained by Ebrahimnejad (2016), is not correct.

7 Exact dummy supply and dummy demand for the existing generalized IVTrFNTP

Ebrahimnejad (2016) solved the generalized IVTrFNTP, to illustrate the proposed method. Since, in the considered generalized IVTrFNTP, total supply $\left\langle\left(110,150,160,180;\ \frac{2}{3}\right),\left(100,140,170,190;\ 1\right)\right\rangle$ is not equal to the total demand $\left\langle\left(90,120,140,200;\ \frac{2}{3}\right),\left(75,105,155,215;\ 1\right)\right\rangle$, Ebrahimnejad applied the proposed method to transform this problem into a balanced generalized IVTrFNTP and claimed that there is a need to add a dummy source having a dummy generalized interval-valued trapezoidal fuzzy (IVTrF) supply $\left\langle\left(25,25,35,75;\ \frac{2}{3}\right),\left(0,25,45,85;\ 1\right)\right\rangle$ as well as a dummy destination having a dummy generalized IVTrF demand $\left\langle\left(45,55,55,55;\ \frac{2}{3}\right),\left(25,60,60,60;\ 1\right)\right\rangle$.

However, as discussed in Section 3, the result of this problem, obtained by Ebrahimnejad (2016), is not correct. In this section, the proposed Mehar method is used to find the correct dummy supply and dummy demand.

Using the proposed Mehar method, the exact dummy supply and dummy demand can be obtained as follows:

$$\sum_{i=1}^{2}\tilde{\tilde{a}}_i = \left\langle \left(\sum_{i=1}^{2} a_{i1}^L, \sum_{i=1}^{2} a_{i2}^L, \sum_{i=1}^{2} a_{i3}^L, \sum_{i=1}^{2} a_{i4}^L;\ \omega^L \right), \left(\sum_{i=1}^{2} a_{i1}^U, \sum_{i=1}^{2} a_{i2}^U, \sum_{i=1}^{2} a_{i3}^U, \sum_{i=1}^{2} a_{i4}^U;\ \omega^U \right) \right\rangle$$

$$= \left\langle \left(110,150,160,180;\ \frac{2}{3} \right), \left(100,140,170,190;\ 1 \right) \right\rangle$$

and $$\sum_{j=1}^{3}\tilde{\tilde{b}}_j = \left\langle \left(\sum_{j=1}^{3} b_{j1}^L, \sum_{j=1}^{3} b_{j2}^L, \sum_{j=1}^{3} b_{j3}^L, \sum_{j=1}^{3} b_{j4}^L;\ \omega^L \right), \left(\sum_{j=1}^{3} b_{j1}^U, \sum_{j=1}^{3} b_{j2}^U, \sum_{j=1}^{3} b_{j3}^U, \sum_{j=1}^{3} b_{j4}^U;\ \omega^U \right) \right\rangle$$

$$= \left\langle \left(90,120,140,200;\frac{2}{3} \right), \left(75,105,155,215;1 \right) \right\rangle.$$

Case 3 of the proposed Mehar method is satisfied. Therefore, according to Case 3 of the proposed Mehar method,

$$A_{33}^U = A_{33}^L + \max\left\{ 0, \left(\left(\sum_{j=1}^{3} b_{j3}^U - \sum_{j=1}^{3} b_{j3}^L \right) - \left(\sum_{i=1}^{2} a_{i3}^U - \sum_{i=1}^{2} a_{i3}^L \right) \right) \right\}$$
$$= 20 + \max\{0, 15 - 10\} = 25,$$

$$A_{34}^L = A_{33}^U + \max\left\{ 0, \left(\left(\sum_{j=1}^{3} b_{j4}^L - \sum_{j=1}^{3} b_{j3}^U \right) - \left(\sum_{i=1}^{2} a_{i4}^L - \sum_{i=1}^{2} a_{i3}^U \right) \right) \right\}$$
$$= 25 + \max\{0, 45 - 10\} = 60,$$

$$A_{34}^U = A_{34}^L + \max\left\{ 0, \left(\left(\sum_{j=1}^{3} b_{j4}^U - \sum_{j=1}^{3} b_{j4}^L \right) - \left(\sum_{i=1}^{2} a_{i4}^U - \sum_{i=1}^{2} a_{i4}^L \right) \right) \right\}$$
$$= 60 + \max\{0, 15 - 10\} = 65,$$

$$B_{41}^U = \max\left\{ 0, \left(\sum_{i=1}^{2} a_{i1}^U - \sum_{j=1}^{3} b_{j1}^U \right) \right\}$$
$$= \max\{0, 100 - 75\} = 25,$$

$$B_{41}^L = B_{41}^U + \max\left\{0, \left(\left(\sum_{i=1}^{2} a_{i1}^L - \sum_{i=1}^{2} a_{i1}^U\right) - \left(\sum_{j=1}^{3} b_{j1}^L - \sum_{j=1}^{3} b_{j1}^U\right)\right)\right\},$$
$$= 25 + \max\{0, 10 - 15\} = 25,$$
$$B_{42}^U = B_{41}^L + \max\left\{0, \left(\left(\sum_{i=1}^{2} a_{i2}^U - \sum_{i=1}^{2} a_{i1}^L\right) - \left(\sum_{j=1}^{3} b_{j2}^U - \sum_{j=1}^{3} b_{j1}^L\right)\right)\right\}$$
$$= 25 + \max\{0, 30 - 15\} = 40,$$

$$B_{42}^L = B_{42}^U + \max\left\{0, \left(\left(\sum_{i=1}^{2} a_{i2}^L - \sum_{i=1}^{2} a_{i2}^U\right) - \left(\sum_{j=1}^{3} b_{j2}^L - \sum_{j=1}^{3} b_{j2}^U\right)\right)\right\}$$
$$= 40 + \max\{0, 10 - 15\} = 40,$$

$$B_{43}^L = B_{42}^L + \max\left\{0, \left(\left(\sum_{i=1}^{2} a_{i3}^L - \sum_{i=1}^{2} a_{i2}^L\right) - \left(\sum_{j=1}^{3} b_{j3}^L - \sum_{j=1}^{3} b_{j2}^L\right)\right)\right\}$$
$$= 40 + \max\{0, 10 - 20\} = 40,$$

$$B_{43}^U = B_{43}^L + \max\left\{0, \left(\left(\sum_{i=1}^{2} a_{i3}^U - \sum_{i=1}^{2} a_{i3}^L\right) - \left(\sum_{j=1}^{3} b_{j3}^U - \sum_{j=1}^{3} b_{j3}^L\right)\right)\right\}$$
$$= 40 + \max\{0, 10 - 15\} = 40,$$

$$B_{44}^L = B_{43}^U + \max\left\{0, \left(\left(\sum_{i=1}^{2} a_{i4}^L - \sum_{i=1}^{2} a_{i3}^U\right) - \left(\sum_{j=1}^{3} b_{j4}^L - \sum_{j=1}^{3} b_{j3}^U\right)\right)\right\}$$
$$= 40 + \max\{0, 10 - 45\} = 40,$$
$$B_{44}^U = B_{44}^L + \max\left\{0, \left(\left(\sum_{i=1}^{2} a_{i4}^U - \sum_{i=1}^{2} a_{i4}^L\right) - \left(\sum_{j=1}^{3} b_{j4}^U - \sum_{j=1}^{3} b_{j4}^L\right)\right)\right\}$$
$$= 40 + \max\{0, 10 - 15\} = 40.$$

Therefore, the dummy fuzzy supply

$$\left\langle \left(A_{(m+1)1}^L, A_{(m+1)2}^L, A_{(m+1)3}^L, A_{(m+1)4}^L;\ \omega^L\right), \left(A_{(m+1)1}^U, A_{(m+1)2}^U, A_{(m+1)3}^U, A_{(m+1)4}^U;\ \omega^U\right) \right\rangle$$
$= \left\langle \left(5, 10, 20, 60;\ \frac{2}{3}\right), (0, 5, 25, 65;\ 1) \right\rangle$ and the dummy fuzzy demand

$$\left\langle \left(B^L_{(n+1)1}, B^L_{(n+1)2}, B^L_{(n+1)3}, BF^L_{(n+1)4};\ \omega^L \right), \left(B^U_{(n+1)1}, B^U_{(n+1)2}, B^U_{(n+1)3}, B^U_{(n+1)4};\ \omega^U \right) \right\rangle$$
$$= \left\langle \left(25, 40, 40, 40;\ \frac{2}{3} \right), (25, 40, 40, 40;\ 1) \right\rangle.$$

8 Exact results of the existing generalized IVTrFNTP

To find the solution of the generalized IVTrFNTP, presented in Ebrahimnejad (2016) has solved a crisp linear programming problem having 56 equality constraints. Out of these 56 equality constraints, 8 constraints correspond to dummy supply and 8 constraints correspond to dummy demand.

However, as discussed in Section 3, the dummy supply and dummy demand, obtained by Ebrahimnejad (2016), is not correct. Therefore, the result of this problem, obtained by Ebrahimnejad (2016), is not correct.

To find the exact result of this problem, the equality constraints C_1 of the existing crisp linear programming problem have been replaced by the constraints C_2.

$$\left.\begin{array}{c}
x^L_{31,1} + x^L_{32,1} + x^L_{33,1} + x^L_{34,1} = 25, \\
x^L_{31,2} + x^L_{32,2} + x^L_{33,2} + x^L_{34,2} = 25, \\
x^L_{31,3} + x^L_{32,3} + x^L_{33,3} + x^L_{34,3} = 35, \\
x^L_{31,4} + x^L_{32,4} + x^L_{33,4} + x^L_{34,4} = 75, \\
x^U_{31,1} + x^U_{32,1} + x^U_{33,1} + x^U_{34,1} = 0, \\
x^U_{31,2} + x^U_{32,2} + x^U_{33,2} + x^U_{34,2} = 25, \\
x^U_{31,3} + x^U_{32,3} + x^U_{33,3} + x^U_{34,3} = 45, \\
x^U_{31,4} + x^U_{32,4} + x^U_{33,4} + x^U_{34,4} = 85, \\
x^L_{14,1} + x^L_{24,1} + x^L_{34,1} = 45, \\
x^L_{14,2} + x^L_{24,2} + x^L_{34,2} = 55, \\
x^L_{14,3} + x^L_{24,3} + x^L_{34,3} = 55, \\
x^L_{14,4} + x^L_{24,4} + x^L_{34,4} = 55, \\
x^U_{14,1} + x^U_{24,1} + x^U_{34,1} = 25, \\
x^U_{14,2} + x^U_{24,2} + x^U_{34,2} = 60, \\
x^U_{14,3} + x^U_{24,3} + x^U_{34,3} = 60, \\
x^U_{14,4} + x^U_{24,4} + x^U_{34,4} = 60.
\end{array}\right\} (C_1)$$

$$\left.\begin{aligned}
x^L_{31,1}+x^L_{32,1}+x^L_{33,1}+x^L_{34,1}&=5,\\
x^L_{31,2}+x^L_{32,2}+x^L_{33,2}+x^L_{34,2}&=10,\\
x^L_{31,3}+x^L_{32,3}+x^L_{33,3}+x^L_{34,3}&=20,\\
x^L_{31,4}+x^L_{32,4}+x^L_{33,4}+x^L_{34,4}&=60,\\
x^U_{31,1}+x^U_{32,1}+x^U_{33,1}+x^U_{34,1}&=0,\\
x^U_{31,2}+x^U_{32,2}+x^U_{33,2}+x^U_{34,2}&=5,\\
x^U_{31,3}+x^U_{32,3}+x^U_{33,3}+x^U_{34,3}&=25,\\
x^U_{31,4}+x^U_{32,4}+x^U_{33,4}+x^U_{34,4}&=65,\\
x^L_{14,1}+x^L_{24,1}+x^L_{34,1}&=25,\\
x^L_{14,2}+x^L_{24,2}+x^L_{34,2}&=40,\\
x^L_{14,3}+x^L_{24,3}+x^L_{34,3}&=40,\\
x^L_{14,4}+x^L_{24,4}+x^L_{34,4}&=40,\\
x^U_{14,1}+x^U_{24,1}+x^U_{34,1}&=25,\\
x^U_{14,2}+x^U_{24,2}+x^U_{34,2}&=40,\\
x^U_{14,3}+x^U_{24,3}+x^U_{34,3}&=40,\\
x^U_{14,4}+x^U_{24,4}+x^U_{34,4}&=40.
\end{aligned}\right\}(C_2)$$

On solving the existing crisp linear programming problem (Ebrahimnejad, 2016) with this modification, the obtained exact optimal solution and the optimal cost of the existing generalized IVTrFNTP, presented by Table 1, is

$$\tilde{\tilde{x}}_{11}=\left\langle\left(30,40,40,50;\frac{2}{3}\right),(25,35,45,55;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{12}=\left\langle\left(15,20,20,20;\frac{2}{3}\right),(15,20,20,20;1)\right\rangle,$$

$$\tilde{\tilde{x}}_{13}=\left\langle\left(0,0,0,0;\frac{2}{3}\right),(0,0,0,0;1)\right\rangle,$$
$$\tilde{\tilde{x}}_{14}=\left\langle\left(25,30,30,30;\frac{2}{3}\right),(25,30,30,30;1)\right\rangle$$

$$\tilde{\tilde{x}}_{21}=\left\langle\left(0,0,10,10;\ \frac{2}{3}\right),(0,0,10,10;1)\right\rangle,\ \tilde{\tilde{x}}_{22}=\left\langle\left(0,0,0,0;\ \frac{2}{3}\right),(0,0,0,0;\ 1)\right\rangle,$$

$$\tilde{\tilde{x}}_{23}=\left\langle\left(40,50,50,50;\ \frac{2}{3}\right),(35,45,55,65;\ 1)\right\rangle,\ \tilde{\tilde{x}}_{24}=\left\langle\left(0,10,10,10;\ \frac{2}{3}\right),(0,10,10,10;\ 1\right\rangle,$$
$$\tilde{\tilde{x}}_{31}=\left\langle\left(0,0,0,10;\ \frac{2}{3}\right),(0,0,0,10;\ 1)\right\rangle,\tilde{\tilde{x}}_{32}=\left\langle\left(5,10,20,30;\ \frac{2}{3}\right),(0,5,25,35;\ 1)\right\rangle,$$

$$\tilde{\tilde{x}}_{33} = \left\langle \left(0,0,0,20; \frac{2}{3}\right), (0,0,0,20;\ 1) \right\rangle,\ \tilde{\tilde{x}}_{34} = \left\langle \left(0,0,0,0; \frac{2}{3}\right), (0,0,0,0;\ 1) \right\rangle.$$

9 Conclusion

In this chapter it has been shown that the existing method for transforming an unbalanced generalized IVTrFNTP into a balanced generalized IVTrFNTP is not valid. In addition, a new method (known as the Mehar method) was proposed for the same purpose and it was proved that this method is valid. Furthermore, the exact result of the existing unbalanced generalized IVTrFNTP was obtained.

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Further reading

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